## Conditions for Nondistortion Interrogation of Quantum System

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Under some physical considerations, we present a universal formulation to study the possibility of localizing a quantum object in a given region without disturbing its unknown internal state. When the interaction between the object and probe wave function takes place only once, we prove the necessary and sufficient condition that the object's presence can be detected in an initial state preserving way. Meanwhile, a conditioned optimal interrogation probability is obtained. PACS numbers: 03.65.Ta, 42.50.Ct, 03.67.-a

In 1993, Elitzur and Vaidman proposed the novel concept of interaction-free measurements (IFMs), in which the presence of an absorbing object in a Mach-Zehnder interferometer can be inferred without apparent interaction with the probe photon [1]. As one of the counter-intuitive quantum effects, IFMs have been extensively investigated [2,3,4,5,6,7,8,9]. Aside from their conceptual significance, IFMs are of obvious application interest too. For instance, people applied the idea of IFM to fields such as quantum cryptography [10], quantum preparation [11], interaction-free imaging [12], counterfactual computation [13] and so on.

In most of these applications, it is desired that the internal state of the object be kept unchanged. However, As emphasized by Vaidman [14], IFMs in general are not initial state preserving measurements. In the case of a quantum object characterized by its quantum superposition, extra care needs to be taken to avoid disturbing the initial state of the object [15,16,17].

In this letter we consider a more general situation: suppose there is a quantum "black box" and the task is to determine if a quantum object is in it, without changing the (unknown) initial state of the object. We may call such an interrogation a nondistortion interrogation of a quantum system (abbreviated NIQS). For the purpose of NIQS, we let a quantum probe wave partially go through the black box. If the black box is occupied with an object a coupling interaction will entangle the originally separate systems and make them diverge from their respective free evolutions. As a result, the final interference pattern of the probe wave may be changed. However, under some special conditions, some interference stripes possibly entangle an identity transformation in the object subspace. Once the probe wave is projected on these stripes, it will leave the object state undisturbed while the existence of the object can be deduced. In this work we prove the necessary and sufficient condition for the NIQS and give the conditioned optimal probability for it, under certain physical assumptions. To obtain our theorems the following assumptions are necessary:

- a) S, the quantum system under interrogation (the quantum object in the black box), is a metastable system whose Hilbert space is denoted as  $H_S$  with dimension n. we assume that the state space  $S(H_S)$  is closed under free evolution.
- b) The Hilbert space of the probe system D,  $H_D$ , is composed of two orthogonal subspaces  $H_r$  and  $H_d$ :  $H_D = H_r \oplus H_d$ . The dimension of the space  $H_d(H_r)$  is m(m'). The initial probe wave function can be written as  $|\Psi_{probe}\rangle = \alpha |\Psi_r\rangle + \beta |\Psi_d\rangle$ . Here,  $|\Psi_r\rangle$  and  $|\Psi_d\rangle$  are chosen from the state space  $S(H_r)$  and  $S(H_d)$  respectively. We let  $|\Psi_d\rangle$  go through the black box and interact with S (if the object is in the box), while  $|\Psi_r\rangle$  is under free evolution.
- c) The interaction between D and S is governed by a unitary operator. We assume the time of the interaction is some known t. When the interaction is over, any state driven out of the metastable state space  $S(H_S)$  will quickly decay in an irreversible way to some stable ground state  $|g\rangle$  which is out of  $S(H_S)$ . The decay signal will be registered by some properly arranged sensitive detectors.

In assumption b), the introduction of  $H_r$  might appear to be redundant since it does not interact with S. But it is actually vital for the purpose of NIQS, because the interaction between  $|\Psi_d\rangle$  and  $|\Psi_S\rangle$  changes the interference between  $|\Psi_r\rangle$  and  $|\Psi_d\rangle$  which provides information on what is in the black box. Based on the above assumptions, the following major steps are followed to find out if there is an object in the black box:

- i) Let the probe wave function  $|\Psi_d\rangle$  go through the black box and interact with the quantum object if it is in the box.
- ii) The decay signal detectors are used to register any decay event. If a decay is detected, we conclude that the quantum object was in the black box but its initial state is destroyed. Otherwise, go to the next step. This is equivalent to a partial projection measurement on the whole system  $\varrho_{tot}$ :

$$\varrho_{out} = P\rho_{tot}P + P_{\perp}\varrho_{tot}P_{\perp}.\tag{1}$$

where the operators P and  $P_{\perp}$  refer to unity operators of Hilbert space  $H_S \otimes H_D$  and its complementary space  $\overline{H_S \otimes H_D}$  respectively.

iii)Perform a Von Neumann measurement on the probe wave function (in  $H_D$ ). Possible measurement outcomes are characterized by some projectors in an orthogonal projector set O.

In step iii), we keep in mind that if nothing is in the black box the probe will be in some definite final state corresponding to the free evolution of the initial state. We designate the projector to that state as  $P_e$ . If a successful interrogation of the object can be done, the probe will end up in some different state. For the consideration of universality we require that the probability of successful nondistortion interrogations of the quantum object be independent on its (unknown) initial state.

Let the initial state of the probe wave function be  $|\Psi_{probe}\rangle = \alpha |\Psi_r\rangle + \beta |\Psi_d\rangle$ , where  $|\Psi_r\rangle \in S(H_r)$  and  $|\Psi_d\rangle \in S(H_d)$ . The time evolution of the whole system is as follows:

$$|\Psi_{probe}\rangle |\Psi_{S}\rangle \to \alpha e^{-i/\hbar (H^S + H^D)t} |\Psi_{r}\rangle |\Psi_{S}\rangle + \beta e^{-i/\hbar} \int_{0}^{t} (H^S + H^D + H^I) dt' |\Psi_{d}\rangle |\Psi_{S}\rangle$$
(2)

where  $H^S$  and  $H^D$  are the free Hamiltonian of the systems S and D respectively, and  $H^I$  characterizes the interaction between the two systems. When the black box is empty the interaction Hamiltonian  $H^I$  vanishes and the above process reduces to the free evolution of two separate systems. In contrast, when the black box is occupied with the quantum object,  $|\Psi_d\rangle |\Psi_S\rangle$  will be driven by  $H^I$  and a decay could happen, since any component out of the state space  $S(H_D \otimes H_S)$  will decay in an irreversible way. If no decay signal is detected, the quantum state of the whole system is collapsed into the following (a projection on  $H_S \otimes H_D$ ):

$$\alpha e^{-i/\hbar(H^S + H^D)t} |\Psi_r\rangle |\Psi_S\rangle + \beta (I_d \otimes I_S) e^{-i/\hbar} \int_0^t (H^S + H^D + H^I) dt' |\Psi_d\rangle |\Psi_S\rangle$$

$$= \alpha |\Psi_r'\rangle |\Psi_S'\rangle + \beta (I_d \otimes I_S) e^{-i/\hbar} \int_0^t (H^S + H^D + H^I) dt' e^{i/\hbar(H^S + H^D)t} (I_d \otimes I_S) |\Psi_d'\rangle |\Psi_S'\rangle$$
(3)

where  $I_d$  and  $I_S$  are the unity operators in the Hilbert spaces  $H_d$  and  $H_S$ ,  $\left|\Psi'_{r(d)}\right\rangle = e^{-i/\hbar H^D t} \left|\Psi_{r(d)}\right\rangle$  and  $\left|\Psi'_S\right\rangle = e^{-i/\hbar H^S t} \left|\Psi_S\right\rangle$ . To simplify the future calculations, the above wave function is unnormalized. In Eq(3), we see that the evolution of the system is fully specified by the following operator D:

$$D = (I_d \otimes I_S)e^{-i/\hbar} \int_0^t (H^S + H^D + H^I)dt' e^{i/\hbar (H^S + H^D)t} (I_d \otimes I_S).$$
 (4)

The following theorem provides the necessary condition for NIQS.

Theorem1: The necessary condition that an NIQS can be done is that there exist a pair of vectors  $|\chi\rangle, |\Psi'_d\rangle \in S(H_d)$  which satisfy  $\langle \chi | D | \Psi'_d \rangle = cI_S$ , where  $|c| \leq 1$ .

Proof: When there is no object in the box, the probe ends up in the state  $\alpha |\Psi'_r\rangle + \beta |\Psi'_d\rangle$ . Let  $P_e = (\alpha |\Psi'_r\rangle + \beta |\Psi'_d\rangle)(\alpha^*\langle \Psi'_r| + \beta^*\langle \Psi'_d|) \otimes I_S$ . In presence of the object, the evolution of the probe wave function is modified. At the end of the interrogation, a measurement is done on the probe. To make sure that a successful NIQS can be done, there must exist a projector  $P_I = |\Psi_I\rangle\langle \Psi_I| \otimes I_S$  in the set O which satisfies

$$P_I P_e = 0 (5)$$

$$P_I(\alpha|\Psi_r'\rangle|\Psi_S'\rangle + \beta D|\Psi_d'\rangle|\Psi_S'\rangle) = \Delta|\Psi_I\rangle|\Psi_S'\rangle \tag{6}$$

where  $|\Psi_I\rangle \in S(H_D)$  is some normalized vector of the probe and  $\Delta$  is a nonzero constant. The probability of this outcome is  $|\Delta|^2$ . From Eqs. (5) and (6), we obtain:

$$\langle \Psi_I | D | \Psi_d' \rangle | \Psi_S' \rangle = \left( \frac{\Delta}{\beta} + \langle \Psi_I | \Psi_d' \rangle \right) | \Psi_S' \rangle \tag{7}$$

Let us introduce a wave vector  $|\chi\rangle = I_d |\Psi_I\rangle$ , the projection of  $|\Psi_I\rangle$  on  $H_d$ . Since  $\Delta$  and  $\beta$  are nonzero we deduce that  $||\chi\rangle|| \neq 0$ . Hence, we have

$$\langle \chi | D | \Psi_d' \rangle | \Psi_S' \rangle = \left( \frac{\Delta}{\beta} + \langle \chi | \Psi_d' \rangle \right) | \Psi_S' \rangle = c | \Psi_S' \rangle \tag{8}$$

where  $c = \frac{\Delta}{\beta} + \langle \chi | \Psi'_d \rangle$ . Since  $| \Psi'_S \rangle$  is arbitrary the following must be satisfied:

$$\langle \chi | D | \Psi_d' \rangle = cI_S.$$
 (9)

In the most general case, it is a nontrivial task to determine whether two vectors  $|\chi\rangle$  and  $|\Psi_d'\rangle$  satisfying (9) exist, for a given D. We give some concrete discussion in the reference [18].

To find out the sufficient condition for the NIQS the operator D needs to be studied further. Under the assumption that Eq(9) holds we may define the operators  $Q^{(i)} = Tr_S[D|\Psi_d'\rangle|i\rangle\langle i|\langle\Psi_d'|D^+]$ . Here,  $\{|i\rangle, i=1,...,n\}$  is a set of orthogonal bases in the Hilbert space  $H_S$ . In the Hilbert space  $H_d$ , the kernel space of the operator  $Q^{(i)}$  is denoted as  $K_i$ . The intersection of all the *n* kernel spaces is  $K = \bigcap_{i=1}^n K_i$ . We denote the dimension of the space  $\overline{K}$ , which is the complementary space of K in  $H_d$ , by  $l(l \leq m)$ . We pick up some set of orthonormal states  $\{|\chi\rangle, |\chi_1\rangle, ..., |\chi_{l-1}\rangle\}$ spanning the space  $\overline{K}$ . Then, Eq(9) can be expressed in the following alternative way:

$$D|\Psi'_{d}\rangle|\Psi'_{S}\rangle = c|\chi\rangle|\Psi'_{S}\rangle + \sum_{j=1}^{l-1}|\chi_{j}\rangle|m_{S(j)}\rangle.$$
(10)

Here,  $|m_{S(j)}\rangle = \langle \chi_j | D | \Psi'_d \rangle | \Psi'_S \rangle$ . By making use of Eq(10) we may provide our main result.

Theorem 2: The necessary and sufficient condition for the NIQS is that Eq(10) holds and  $|\Psi'_d\rangle - c|\chi\rangle$  is linearly independent of the state set  $\{|\chi_j\rangle; j=1,...,l-1\}$ .

Proof: We have proved that Eq(10) is the necessary condition. If Eq(10) holds, in Hilbert space  $H_S \otimes H_D$  the final state of the whole system is:

$$|\Psi_{probe}\rangle |\Psi_{S}\rangle \to \alpha |\Psi_{r}'\rangle |\Psi_{S}'\rangle + \beta c |\chi\rangle |\Psi_{S}'\rangle + \beta \sum_{j=1}^{l-1} |\chi_{j}\rangle |m_{S(j)}\rangle.$$
 (11)

Considering Eqs. (5) and (6), we have:

$$\langle \Psi_I | \left( \alpha | \Psi_r' \rangle + \beta | \Psi_d' \rangle \right) = 0 \tag{12}$$

$$\langle \Psi_I | \chi_i \rangle = 0 \tag{13}$$

$$\langle \Psi_I | (\alpha | \Psi_r') + c\beta | \chi \rangle) = \Delta. \tag{14}$$

Subtracting Eq(14) from Eq(12) we obtain

$$\langle \Psi_I | \beta (|\Psi_I' \rangle - c | \chi \rangle) = -\Delta. \tag{15}$$

 $\Delta \neq 0$  requires that  $|\Psi_d'\rangle - c |\chi\rangle$  be linearly independent of the set of vectors  $\{|\chi_j\rangle; j=1,...,l-1\}$ . Now if Eq(10) holds and  $|\Psi_d'\rangle - c |\chi\rangle$  is linearly independent of the state set  $\{|\chi_j\rangle; j=1,...,l-1\}$ , we may prove the sufficient condition by constructing a projector  $P_I$  satisfying Eq(5) and Eq(6). We do this by using the Schmidt orthogonalization process. First we define the state set N consisting of l+1 normalized vectors  $\{\alpha \mid \Psi_r' \rangle + \beta \mid \Psi_d' \rangle$ ,  $\gamma \left(\alpha \mid \Psi_r' \rangle + c\beta \mid \chi \rangle\right)$ ,  $|\chi_j \rangle$ ;  $j=1,...,l-1\}$ , where  $\gamma = \frac{1}{\|\alpha \mid \Psi_r' \rangle + c\beta \mid \chi \rangle\|}$  is the normalization coefficient for  $\alpha \mid \Psi_r' \rangle + c\beta \mid \chi \rangle$  $c\beta |\chi\rangle$ . We assume that  $\alpha \neq 0$ . Since  $|\chi_1\rangle$ , ...  $|\chi_{l-1}\rangle$  and  $|\chi\rangle$  are orthogonal to each other and  $|\Psi'_d\rangle - c|\chi\rangle$  is linearly independent of  $\{|\chi_j\rangle; j=1,...,l-1\}$ , we may deduce that all vectors in the state set N are linearly independent. To make an orthonormal set out of N, we let the first l-1 vectors be  $\{|\chi_j\rangle; j=1,...,l-1\}$ . We then calculate the lth vector using  $\alpha |\Psi'_r\rangle + \beta |\Psi'_d\rangle$ :  $|\widetilde{\Psi}\rangle = \gamma' \left(\alpha |\Psi'_r\rangle + \beta |\Psi'_d\rangle - \sum_{i=1}^{l-1} \langle \chi_i | \left(\alpha |\Psi'_r\rangle + \beta \Psi'_d\rangle\right) |\chi_i\rangle\right)$ , where  $\gamma'$  is the normalization coefficient. Similarly, we can obtain the (l+1)th vector  $|\Psi_I\rangle = \gamma'' \left( \gamma \left( \alpha \left| \Psi_r' \right\rangle + c\beta \left| \chi \right\rangle \right) - \left\langle \widetilde{\Psi} \right| \gamma \left( \alpha \left| \Psi_r' \right\rangle + c\beta \left| \chi \right\rangle \right) \left| \widetilde{\Psi} \right\rangle \right)$ .

It is then obvious that the projector  $P_I = |\Psi_I\rangle\langle\Psi_I| \otimes I_S$  satisfies Eqs(5) and (6). We thus complete our proof.

To clarify the connotation of the above theorems let us consider the following physical picture of NIQS: the quantum object in the black box can be seen as a "scattering object" corresponding to the probe wave. Due to the interaction with the object the probe wave will change its initial coherence. On the other hand, the quantum object is also affected by the probe wave. Thus, each scattering wave component entangles different evolution of the object. If we know all the information on the evolution of the composite system it possibly allows us to choose a proper probe wave such that a successful scattering wave component can be produced. For the purpose of NIQS this component should correspond to the free evolution of the object, at the same time, be orthogonal to any other scattering wave components. (The form of this component can be obtained by using Schmidt orthogonalization steps outlined in the proof of theorem 2.) Once this scattering wave component is registered by a detector we may deduce the existence of the object in the internal preserving way. In fact, theroem 2 provides the necessary and sufficient condition for the existence of this scattering wave component.

Let us denote the Hilbert space spanned by  $\{|\Psi'_r\rangle, |\chi\rangle, |\chi_1\rangle, ... |\chi_{l-1}\rangle\}$  as  $H_{l+1}$  and its complementary space in  $H_D$  as  $H_{l+1}^{\perp}$ . For the success probability of an NIQS we have the following corollary:

Corollary 1:  $P_I$  obtained by using Schmidt orthogonalization process in  $H_{l+1}$  (as outlined above) maximizes the success probability  $|\Delta|^2$  for a given  $\alpha$ .

Proof: Let  $I_{l+1}$  and  $I_{l+1}^{\perp}$  be the unity operators of  $H_{l+1}$  and  $H_{l+1}^{\perp}$  respectively. Any Von Neumann projector characterizing a successful nondistortion interrogation output can be written as  $P'_{I} = |\Psi'_{I}\rangle\langle\Psi'_{I}| \otimes I_{S}$  ( $P'_{I}$  satisfies (5), (6)). Furthermore, we have the following relation:

$$P'_{I} = (I_{l+1} \otimes I_{S}) P'_{I}(I_{l+1} \otimes I_{S}) + (I_{l+1}^{\perp} \otimes I_{S}) P'_{I}(I_{l+1}^{\perp} \otimes I_{S})$$

$$= dP_{I} + (I_{l+1}^{\perp} \otimes I_{S}) P'_{I}(I_{l+1}^{\perp} \otimes I_{S})$$
(16)

where  $|d| \leq 1$ . The second equality is because the projector  $P_I$  satisfying the NIQS conditions (Eqs(12)-(14)) in  $H_{l+1} \otimes H_S$  is unique. On the other hand, the second term in the last line will never contribute to the projection probability when  $P_I'$  operates on  $\alpha |\Psi_T'\rangle |\Psi_S'\rangle + \beta D|\Psi_d'\rangle |\Psi_S'\rangle$ . Hence, the success probability of  $P_I'$  is:

$$\Pr ob(\alpha)_{P_I'} = \left| \left\langle \Psi_I' \right| \left( \alpha \left| \Psi_r' \right\rangle + c\beta \left| \chi \right\rangle \right) \right|^2$$

$$= \left| d \right|^2 \left| \left\langle \Psi_I \right| \left( \alpha \left| \Psi_r' \right\rangle + c\beta \left| \chi \right\rangle \right) \right|^2 \le \Pr ob(\alpha)_{P_I}$$
(17)

The above corollary shows that the maximal success probability for the NIQS process is just the probability of this successful scattering wave component when the original probe wave is given. We note that the vectors  $|\Psi'_d\rangle$  and  $|\chi\rangle$  satisfying Eq(9) may not be unique. However, once  $|\chi\rangle$  and  $|\Psi'_d\rangle$  are chosen we may obtain the optimal success probability for these two vectors, by Schmidt orthogonalization steps .

Corollary 2: Under the condition that the wave function  $|\Psi'_d\rangle$  and  $|\chi\rangle$  are given the optimal success probability of the NIQS is as follows:

$$P_{opt} = \max_{|\alpha| \in [0,1)} \Pr{ob(\alpha)_{P_I}}.$$
(18)

To elucidate our result in a more concrete way, let us consider an example recently investigated [15,16,17]. As in Fig. 1, the atom is prepared in an arbitrary superposition of the two metastable states  $|m_{+}\rangle$  and  $|m_{-}\rangle$ . By absorbing a + or - (circularly) polarized photon the atom can make a transition to the excited state  $|e\rangle$ . It then decays rapidly to the ground state  $|g\rangle$  in an irreversible way. Now, without disturbing the initial state of the atom we want to find out if it is in the black box located in the lower arm of the Mach-Zehnder interferometer. To do that, we use an x polarized photon  $\frac{1}{\sqrt{2}}(|l_+\rangle + |l_-\rangle)$  as the probe and direct it into the interferometer. The amplitude transmission and reflection coefficients of the two beam splitters are  $(\alpha, \beta)$  and  $(\beta, \alpha)$  respectively so that the photon exits from the upper port of  $PBS_2$  with certainty when the atom is not in the black box.  $PBS_1$  changes the state of the incident photon to  $|\Psi_{probe}\rangle = \frac{\alpha}{\sqrt{2}} (|u_{+}\rangle + |u_{-}\rangle) + \frac{\beta}{\sqrt{2}} (|l_{+}\rangle + |l_{-}\rangle)$ , where l(u) refers to the lower (upper) optical path and +(-) refers to the polarization of the photon. After the first beam splitter, the photon wave function is in a superposition of the upper and lower branches. These correspond to  $H_r$  and  $H_d$  in our formulation. Here  $H_r$ ,  $H_d$  are two dimensional subspaces with base vectors  $|u_{\pm}\rangle$  and  $|l_{\pm}\rangle$ .  $H_S$ , the metastable state space of the atom, is a two dimensional space spanned by  $|m_{+}\rangle$  and  $|m_{-}\rangle$ . The interaction between  $H_d$  and  $H_S$  and the afterward dissipation is characterized by the operator  $D=|l_{+}\rangle\,|m_{-}\rangle\,\langle m_{-}|\,\langle l_{+}|+|l_{-}\rangle\,|m_{+}\rangle\,\langle m_{+}|\,\langle l_{-}|$ . We may further deduce that the space  $\overline{K}$  is just  $H_{d}$ . Thus, when choosing  $|\Psi'_{d}\rangle=|\chi\rangle=\frac{1}{\sqrt{2}}(|l_{+}\rangle+|l_{-}\rangle)$  and  $|\chi_{1}\rangle=\frac{1}{\sqrt{2}}(|l_{+}\rangle-|l_{-}\rangle)$  we find  $\langle\chi\,|D|\,\Psi'_{d}\rangle=|m_{-}\rangle$  $\frac{1}{2}\left(\left|m_{-}\right\rangle\left\langle m_{-}\right|+\left|m_{+}\right\rangle\left\langle m_{+}\right|\right)=\frac{1}{2}I_{S} \text{ and } \left|\Psi_{d}^{\prime}\right\rangle-\frac{1}{2}\left|\chi\right\rangle \text{ is linearly independent of the state vector } \left|\chi_{1}\right\rangle.$  Therefore, the sufficient and necessary condition for NIQS is satisfied. Following the Schmidt orthogonalization process we can obtain the projector  $P_I$ :  $P_I = \left(\frac{\beta^*}{\sqrt{2}}(|u_+\rangle + |u_-\rangle) - \frac{\alpha^*}{\sqrt{2}}(|l_+\rangle + |l_-\rangle)\right)\left(\frac{\beta}{\sqrt{2}}(\langle u_+| + \langle u_-|) - \frac{\alpha}{\sqrt{2}}(\langle l_+| + \langle l_-|)\right)\otimes I_S$ . Here, the actual measurement of the probe photon is performed after  $PBS_2$  with the projector  $P_I' = UP_IU^{\dagger} =$ Thus, the success probability is  $\Pr{ob(\alpha)} = \frac{1}{4} |\alpha|^2 |\beta|^2$ . When  $|\alpha| = |\beta| = \frac{1}{\sqrt{2}}$  the optimal probing probability is obtained:  $P_{opt} = \frac{1}{16}$ . So, when the lower detector fires we can deduce that the atom is in the black box, with success probability 1/16. This example has just been investigated by Pötting et. al [15].

FIG. 1. A nondistortion interrogation of an atom prepared in an arbitrary superposition of the metastable states  $|m_{+}\rangle$  and  $|m_{-}\rangle$ , which are coupled to the excited state  $|e\rangle$  through + or - (circularly) polarized photons. An x polarized photon is used as the probe. The black box is located in the lower optical path.

In our scheme we assume that the interaction between the object and probe system occurs only once. It has been shown that under certain conditions a higher efficiency can be obtained in an iterative way [2,3,5,8,17]. Some results on this particular scheme have been obtained in ref [18].

It should be emphasized that the significance of NIQS lies in the coherence of the detected quantum state being preserved in the Hilbert space  $H_S$ . In other words, quantum information in Hilbert space  $H_S$  will not be contaminated by the probing process. This way to manipulate quantum objects is of potential application in the recently developed quantum information science. Since the detected system need not be restricted to pure states we may also cast our interests on mixed states. As pointed out by Pötting et. al [15] the nondistortion interrogation provides a tool to monitor a subsystem in a many-particle system without destroying the entanglement between the particles.

In this letter we have studied the process of NIQS under some physical assumptions. We proved the necessary and sufficient condition for NIQS process in our formulation. We obtained the optimal success probability of NIQS when the state vectors  $|\Psi'_d\rangle$  and  $|\chi\rangle$  are given. We also showed that our results apply to IFMs which is a special case of the problem we discussed. As a novel method to manipulate quantum systems NIQS may be applied in future quantum information processing.

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